Isospin structure of an E2 transition matrix element in ^{27}Al and ^{27}Si

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derive

$$\frac{d^2S}{dt^2} = -\frac{2K^2}{C}\int_a^b \left\{\frac{1}{T^2}\left(\frac{\partial^2 T}{\partial x^2}\right)^2 - \frac{1}{T^3}\left(\frac{\partial T}{\partial x}\right)^2 \frac{\partial^2 T}{\partial x^2}\right\} dx \tag{8}$$

on making use of conditions (4). To prove that the integral in equation (8) is positive we use Schwarz's inequality in the form

$$\int_a^b \{f(x)\}^2 \, dx \int_a^b \{g(x)\}^2 \, dx \geqslant \left| \int_a^b f(x) g(x) \, dx \right|^2$$

where $f(x) = T^{-1}(\partial^2 T/\partial x^2)$ and $g(x) = T^{-2}(\partial T/\partial x)^2$. This gives

$$\int_a^b \frac{1}{T^2}\left(\frac{\partial^2 T}{\partial x^2}\right)^2 dx \int_a^b \frac{1}{T^4}\left(\frac{\partial T}{\partial x}\right)^4 dx \geqslant \left| \int_a^b \frac{1}{T^3}\left(\frac{\partial T}{\partial x}\right)^2 \frac{\partial^2 T}{\partial x^2} dx \right|^2. \tag{9}$$

Now, on integrating 'by parts', and using equations (4), it is readily shown that

$$\int_a^b \frac{1}{T^3}\left(\frac{\partial T}{\partial x}\right)^2 \frac{\partial^2 T}{\partial x^2} dx = \int_a^b \frac{1}{T^4}\left(\frac{\partial T}{\partial x}\right)^4 dx \tag{10}$$

and, since the right hand side of this equality is clearly positive, it follows from inequality (9) that

$$\int_a^b \frac{1}{T^2}\left(\frac{\partial^2 T}{\partial x^2}\right)^2 dx \geqslant \int_a^b \frac{1}{T^3}\left(\frac{\partial T}{\partial x}\right)^2 \frac{\partial^2 T}{\partial x^2} dx.$$

Thus, we see from equation (8) that the inequality (2) holds.

The result proved here suggests that further work may well be justified on the possible extension of inequality (1) to apply to isolated systems well away from equilibrium.

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Isospin structure of an *E*2 transition matrix element in ^{27}Al and ^{27}Si

Abstract. *E*2/*M*1 mixing ratios have been measured for transitions from the second ($3/2^+$), third ($7/2^+$) and fourth ($5/2^+$) excited states of ^{27}Si. A discrepancy in the magnitudes of the mixing ratios for the $5/2^+ \rightarrow$ ground state transition in ^{27}Al and ^{27}Si is confirmed, and is used to estimate the ratio of the isovector to the isoscalar components of the *E*2 matrix element for the transition.

General considerations (Warburton and Weneser 1969) regarding $\Delta T = 0$ electromagnetic transitions of reasonable strength predict that *E*2 matrix elements should be largely isoscalar in character, and *M*1 largely isovector. Comparison of the

Table 1. Measured $E2/M1$ mixing ratios for transitions from the second, third and fourth excited states of ^{27}Si, with the corresponding values for ^{27}Al for comparison

Transition†	da Silva *et al.*	Lewis *et al.*	This work	Weighted mean	^{27}Al (Smulders *et al.*)
$\frac{3}{2}^+(1) \to \frac{5}{2}^+(1)$	−0·26 ±0·01	−0·36 ±0·03	−0·27 ±0·09	−0·31 ±0·02‡	+0·37 ±0·03
$\frac{7}{2}^+(1) \to \frac{5}{2}^+(1)$		+0·43 ±0·04	+0·33 ±0·04	+0·38 ±0·03	−0·41 ±0·02
$\frac{5}{2}^+(2) \to \frac{3}{2}^+(1)$		+0·06 ±0·04	+0·09 ±0·03	+0·08 ±0·02	−0·09 ±0·03
$\frac{5}{2}^+(2) \to \frac{5}{2}^+(1)$		+0·5 ±0·2	+0·38 ±0·08	+0·40 ±0·07	−0·09 ±0·03

† The figures in brackets are ordinal numbers for levels of a given spin.

‡ Because different methods were apparently used to estimate the errors in the previous measurements of δ, the mean value has been evaluated in this case by setting the error of da Silva *et al.* equal to that of Lewis *et al.*

absolute strengths of corresponding transitions in isobaric triads (Bini *et al.* 1970, Schulz and Shapiro 1970) gives results consistent with the conclusion for $E2$ matrix elements. For weak ('noncollective') $E2$ transitions, in which the effective isoscalar charge (e_p+e_n) no longer predominates over the effective isovector charge (e_p-e_n), it is harder to extract the relative isovector and isoscalar components from the absolute strengths, since the experimental errors on the lifetimes, and on branching ratios for weak transitions, tend to be prohibitively large.

In such cases, the precise measurement of $E2/M1$ amplitude mixing ratios offers an alternative approach which should yield also the relative *phases* of the isospin components. For corresponding $\Delta T = 0$ transitions in mirror nuclei, the mixing ratios should be equal in magnitude if the transitions are strong, and should become increasingly different as the transition strengths get weaker. Glaudemans and van der Leun (1971) found, however, no significant departure from equality in 20 mirror pairs covering a wide range of $E2$ and $M1$ strengths in the sd shell. In this letter we report a case in which a significant difference is observed, and estimate the isovector–isoscalar content of an $E2$ matrix element which would give rise to such a difference.

The only reported measurement, by Lewis *et al.* (1967), of the $E2/M1$ mixing ratio δ for the transition from the fourth excited state (2·65 MeV, $5/2^+$) to ground ($5/2^+$) in ^{27}Si (an approximately 25% branch) indicates a magnitude approximately five times that for the corresponding transition in ^{27}Al (Endt and van der Leun 1967), which is known (Smulders *et al.* 1968) to have an $E2$ strength ($\simeq$ 0·1 Wu) (Wu = Weisskopf unit) smaller than any of the transitions reviewed by Glaudemans and van der Leun. Since the difference between the two values is less than two standard deviations, we have remeasured the ^{27}Si mixing ratio to a precision comparable with that of the ^{27}Al value; our results also include transitions from the second (0·96 MeV, $3/2^+$) and third (2·17 MeV, $7/2^+$) levels.

Angular correlations of γ rays produced in the reaction ^{28}Si(τ, $\alpha\gamma$) ^{27}Si were measured at a bombarding energy $E_\tau = 15{\cdot}0$ MeV; the apparatus and methods of analysis are described by Main *et al.* (1970). The results for all the transitions studied are compared with the previous measurements in table 1, which also summarizes corresponding data for ^{27}Al. All signs quoted are in accordance with the phase conventions of Rose and Brink (1967).

The correlation of the 2·65 MeV ($5/2^+ \rightarrow 5/2^+$) γ ray is shown in figure 1, together with the χ^2 plot used to deduce the result $\delta = 0{\cdot}38 \pm 0{\cdot}08$. This value is in agreement with the earlier measurement; the weighted mean value gives, for the ratio of the two mixing ratios, $\delta_{Al}/\delta_{Si} = -0{\cdot}23 \pm 0{\cdot}09$.

The mean lifetime of the 2·65 MeV level has recently been measured by Hutcheon *et al.* (1971) as 35 ± 16 fs. The associated $M1$ and $E2$ transition strengths, deduced from this result and the mixing ratio data of table 1, are compared with the

Table 2. Transition strengths, in Weisskopf units, for the decay of the fourth excited ($5/2^+$) state in ^{27}Al and ^{27}Si

Transition	$M1$ strengths		$E2$ strengths	
	^{27}Al	^{27}Si	^{27}Al	^{27}Si
$\frac{5}{2}^+(2) \rightarrow \frac{3}{2}^+(1)$	$0{\cdot}30 \pm 0{\cdot}15$	$0{\cdot}14 \pm 0{\cdot}06$	4 ± 2	$1{\cdot}6 \pm 1{\cdot}1$
$\frac{5}{2}^+(2) \rightarrow \frac{5}{2}^+(1)$	$0{\cdot}02 \pm 0{\cdot}01$	$0{\cdot}010 \pm 0{\cdot}005$	$0{\cdot}09 \pm 0{\cdot}05$	$1{\cdot}3 \pm 0{\cdot}7$

The ^{27}Al results are taken from Smulders *et al.* (1968).

corresponding ^{27}Al data in table 2. The only significant T_3 dependence occurs in the $E2$ part of the $5/2^+ \to 5/2^+$ transition, for which the ^{27}Si value exceeds that for ^{27}Al by a factor 14 ± 11.

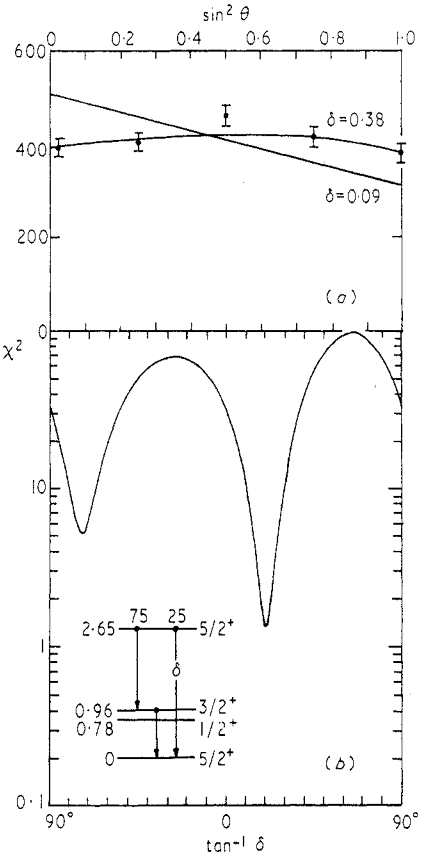

Figure 1. (a) Angular correlation of 2·65 MeV γ rays in coincidence with α particles associated with the 2·65 MeV ($5/2^+$) level in ^{27}Si and detected at angles near 180° to the beam direction. The full curve drawn through the points is the best fit to the data; also shown is the best fit obtained for a mixing ratio equal in magnitude and opposite in sign to the value measured in ^{27}Al. (b) Plot of χ^2 against $\tan^{-1}\delta$ for theoretical fits to the above angular correlation.

An estimate may be obtained from the mixing ratio data of the magnitude and sign of the ratio $V(E2)/S(E2)$, where $S(E2)$ and $V(E2)$ are respectively the isoscalar and isovector parts of the reduced $E2$ matrix element $M(E2)$, by assuming that the $M1$ component is completely independent of T_3 (i.e. $|V(M1)| \gg |S(M1)|$). In that case,

$$-\frac{\delta_{\mathrm{Al}}}{\delta_{\mathrm{Si}}} \simeq \left(\frac{M(E2)}{E_\gamma}\right)_{\mathrm{Al}} \left(\frac{E_\gamma}{M(E2)}\right)_{\mathrm{Si}}$$

$$= \frac{2{\cdot}65}{2{\cdot}73}\,\frac{S(E2)+V(E2)/\sqrt{3}}{S(E2)-V(E2)/\sqrt{3}}.$$

Substituting the measured ratio δ_{A1}/δ_{S1} gives the result $V(E2)/S(E2) = -1{\cdot}07 \pm 0{\cdot}15$. Neglect of any unobserved T_3-dependence in $M(M1)$ must increase the uncertainty of this value, but $V(E2)/S(E2)$ varies only slowly with $V(M1)/S(M1)$ unless $S(M1)$ and $V(M1)$ are comparable in magnitude.

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